6.1 α- and β- LANTHANUM.

Lanthanum at room temperature generally consists of a mixture of two allotropes, face-centered cubic (β-La) and hexagonal (α-La), unless special precautions are taken to obtain one or the other allotrope. The α-β transformation occurs on heating at about 580°K (310°C) and on cooling at about 490°K (220°C). The transformation on cooling $\beta \rightarrow \alpha$ is sluggish and very seldom goes to completion. All of the experimental data, which were analyzed to determine the band structure of lanthanum, are from samples which contain both phases. Thus it is impossible to know the correct value of the density of states of either α or β-La. It is, however, possible to determine the density of states of α-La relative to β-La. This is outlined below.

Suhl and Matthias [30] have given a relationship between the density of states and the superconducting transition temperature, T_s, for a superconductor (*e.g.* lanthanum) containing some paramagnetic impurity atoms (*e.g.* gadolinium). Rewriting their Eqn. (29) as:

$$\frac{dT_s}{dc} = -\frac{\pi^2}{7} J^2 S(S+1) \frac{N(E)}{k} \qquad (13)$$

where c is the concentration of the paramagnetic impurity in a/o, J the exchange energy of the spin coupling between a paramagnetic ion and a conduction electron, S the spin of the paramagnetic ion and k the Boltzmann constant. Since J is quite difficult to evaluate, only an order of magnitude value can be obtained for N(E). But for lanthanum the value of dT_s/dc is known for gadolinium impurities in both α and β-La. If we make the reasonable assumption that J is identical for these two cases, then we see that the term $(\pi^2/7k)J^2 S(S+1)$ is a constant, and that

$$\frac{N(E)_\alpha}{N(E)_\beta} = \left(\frac{dT_s}{dc}\right)_\alpha \Bigg/ \left(\frac{dT_s}{dc}\right)_\beta \qquad (14)$$

where the subscripts α and β designate α-La and β-La respectively. The value of $(dT_c/dc)_\alpha$ is -4.5°K per 1 a/o [31] and $(dT_s/dc)_\beta$ is -5.1°K per 1 a/o [16]. Substituting these values into Eqn. (14) we find $N(E)_\beta = 1.13N(E)_\alpha$, or in other words the density of states of β-La (face-centered cubic) is 13 per cent larger than that of α-La (hexagonal).

6.2 PREDICTION OF HALL COEFFICIENTS.

The Hall coefficients of terbium, holmium and scandium have not been measured, however, it is possible to estimate these values from their measured electronic specific heat constants, γ. If the density of states, N(E), is calculated from γ by using Eqn. (3), we can determine the number of holes in the s band from Fig. 4, which in turn can be used to find NeR_H from Fig. 3. Once NeR_H is known it is a simple matter of calculating the Hall coefficient, R_H. The predicted values of the Hall coefficients (in units of volt-cm/amp-Oe) are:

Tb	-0.11×10^{-12}
Ho	-1.5×10^{-12}
Sc	-0.32×10^{-12}.

It is interesting to note that as the final pages of this paper were being written an abstract [32] of a Russian publication on some transport and magnetic properties of scandium became available. This abstract [32] noted that scandium has a Hall coefficient of -0.30×10^{-12}, which is in very good agreement with the predicted value.

In a similar manner the Hall coefficient of β-La with respect to that of α-La can be estimated from the result shown earlier that $N(E)_\beta = 1.13\, N(E)_\alpha$. It is found that $(R_H)_\beta \simeq 0.4\, (R_H)_\alpha$, and since a sample containing a mixture of both α and β-La phases had a R_H value of -0.8×10^{-12}, it is obvious that the Hall coefficient of β-La will be less negative than that of α-La.